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# Failure of elasto-plastic porous materials subjected to triaxial loading conditions

K. Danas[1], P. Ponte Castañeda[2]

[1] LMS, Ecole Polytechnique, France, kdanas@lms.polytechnique.fr
[2] Department of Mechanical Engineering and Applied Mechanics, University of Pennsylvania, Philadelphia, U.S.A., ponte@seas.upenn.edu

**Résumé** — This work makes use of the recently proposed second-order nonlinear homogenization model (SOM) for (visco)plastic porous materials [1] to study the influence of the Lode parameter and the stress triaxiality on the failure of metallic materials. This model is based on the "second-order" or "generalized secant" homogenization method [2] and is capable of handling general "ellipsoidal" microstructures (i.e., particulate microstructures with more general orthotropic overall anisotropy) and general three-dimensional loading conditions.
**Mots clés** — ductile fracture, Lode parameter, plasticity.

## 1 Introduction

Failure and ductile fracture of metallic materials has received a lot of attention the last sixty years. One of the main reasons for material failure is the presence or/and nucleation of voids and micro-cracks which tend to evolve in volume fraction, shape orientation as a result of the applied loading conditions. For several years, it was believed that the stress triaxiality, denoted here as $X_\Sigma$ and defined as the ratio between the mean stress to the von Mises equivalent or effective deviatoric stress, is the main loading parameter that controls ductile fracture. In particular, large amount of experimental data [3, 4] has shown a monotonic decrease of material ductility with the increase of the stress triaxiality. Nonetheless, recent experimental evidence [5, 6] indicate a substantial decrease of the material ductility with decrease of stress triaxiality and certain shear loading conditions. In these studies, it has been identified that a second loading parameter, the Lode parameter, $L$ (or equivalently Lode angle, $\theta$) controls the ductile fracture mechanism at low stress triaxialities. The Lode parameter is a function of the third invariant of the stress deviator and is used to distinguish between the different shear stress states that can be present in a loading history (see details in the following section). In the present work, we make use of the "second-order" model [1] (SOM) to study the influence of the Lode parameter and the stress triaxiality on ductile failure.

## 2 Modeling and results

We consider a porous material with initially spherical voids subjected to purely triaxial loading conditions with the principal stresses $\sigma_1 = \sigma_{11}$, $\sigma_2 = \sigma_{22}$ and $\sigma_3 = \sigma_{33}$ ($\sigma_{ij} = 0$ for $i \neq j$) being aligned with the laboratory frame axes, $\mathbf{e}^{(1)}$, $\mathbf{e}^{(2)}$ and $\mathbf{e}^{(3)}$, respectively. This allows for the definition of alternative stress measures appropriate for dilatational plasticity, which is the case in the context of porous materials. The three alternative measures are the von Mises equivalent stress (or effective stress), $\sigma_e$, the mean stress, $\sigma_m$, and the third invariant of the stress deviator $J_3$ defined all by

$$\sigma_e = \sqrt{3J_2} = \sqrt{3 s_{ij}\, s_{ij}/2}, \qquad \sigma_m = \sigma_{kk}/3, \qquad J_3 = \det(s_{ij}) = \frac{1}{3} s_{ij}\, s_{ik}\, s_{jk}, \tag{1}$$

where $s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij}$ is the stress deviator. Using these definitions, we evaluate the Lode parameter, $L$, or equivalently the Lode angle, $\theta$, and the stress triaxiality, $X_\Sigma$, by the following expressions

$$L = -\cos 3\theta = \frac{27}{2}\frac{J_3}{\sigma_e^3}, \qquad X_\Sigma = \frac{\sigma_m}{\sigma_e}, \tag{2}$$

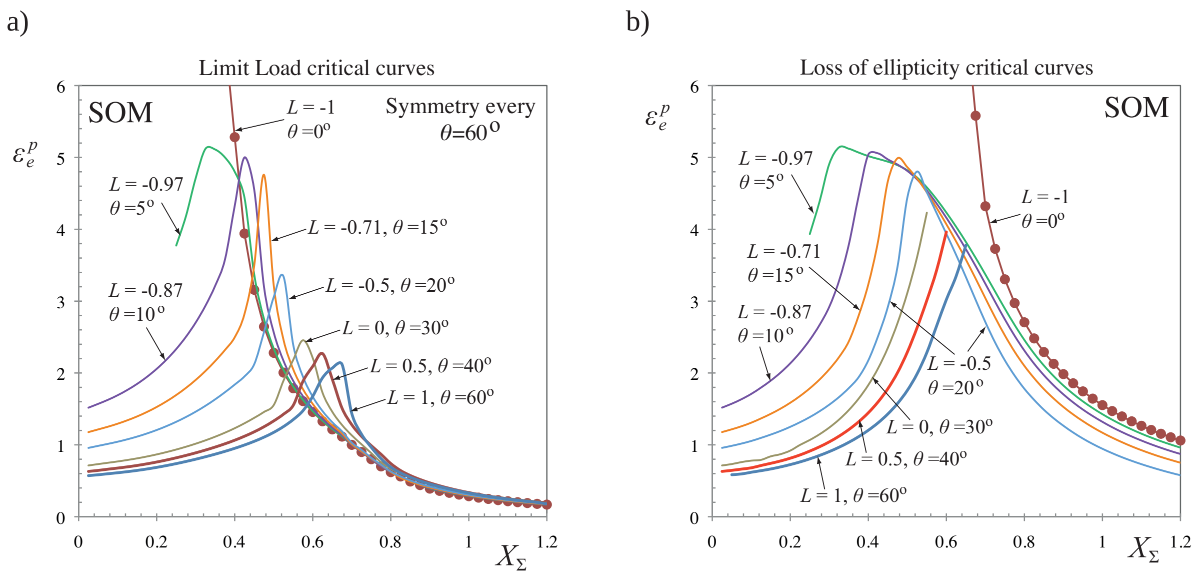


FIGURE 1 – (a) Limit load and (b) loss of ellipticity maps as predicted by the SOM model as a function of the stress triaxiality $X_\Sigma$ and the Lode parameter $L$ (or $\theta$). The limit load locus is identified with the critical equivalent plastic strain $\varepsilon_e^p$ where the hardening rate $H = 0$. The loss of ellipticity locus is identified with the critical equivalent plastic strain $\varepsilon_e^p$ where the constitutive equations lose strong ellipticity leading to localized bifurcated solutions and long-wavelength instabilities. The hardening exponent is $N = 0.1$ and the initial porosity $f_0 = 1\%$.

with $-1 \leq L \leq 1$ (or $0 \leq \theta \leq \pi/3$) and $-\infty < X_\Sigma < +\infty$. The values $\theta = \kappa\pi/3$ ($L = -1$ and $L = +1$) and $\theta = (2\kappa + 1)\pi/6$, with $\kappa$ integer, correspond to axisymmetric shear and simple shear with superimposed hydrostatic pressure when $X_\Sigma$ is nonzero. When $|X_\Sigma| \to \infty$ and $X_\Sigma = 0$ the loading is purely hydrostatic and purely deviatoric, respectively.

Fig. 1 shows SOM maps of the critical equivalent plastic strain $\varepsilon_e^p$ attained when (a) the limit load (i.e., maximum in the $\sigma_e - \varepsilon_e$ curve or equivalently critical hardening rate $H = 0$) and (b) the conditions for localization [7] and loss of ellipticity (LOE) are reached as a function of the stress triaxiality $X_\Sigma$ and the Lode parameter $L$ (or Lode angle $\theta$). We find that the overall strain at localization depends strongly on the Lode parameter as well as on the stress triaxiality. According to the SOM model, these effects are due to the collapse of voids at low stress triaxialities where the initially spherical pores tend to become micro-cracks. This void collapse mechanism leads to the formation of shear or dilation localization bands depending on the value of the Lode parameter. On the other hand, at large stress triaxialities the main mechanism of failure is the increase of porosity, which leads to the overall softening of the porous material. The interplay of these two different mechanisms of localization (i.e., void shape change and porosity growth), leads to sharp, high-ductility corners on the localization locus map.

## Références


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