POROUS POWER-LAW COMPOSITES: YIELD SURFACES AND EVOLUTION OF MICROSTRUCTURE

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1 INTRODUCTION

This work is concerned with the application of the "second-order" nonlinear homogenization procedure of Ponte Castañeda (2002 a,b), denoted as SOM, to generate estimates of the Hashin-Shtrikman type for the effective behavior of two-phase, porous, power-law composites. The SOM is applied to plane-strain nonlinear porous composites weakened by aligned cylindrical (ellipsoidal) voids *randomly* distributed so that the overall symmetry of the composite is transversely isotropic. The microstructure (voids) is allowed to evolve in such a manner that the initially isotropic response of the composite becomes anisotropic. This is due to the change in shape and orientation of the voids in the plane $1-2$. The change in the shape of the void is identified with the evolution of the aspect ratio of the void, λ, while the change in the orientation of the void is given by the change in the angle, ψ, formed between the largest principal axis of the void and the laboratory-frame axis 1, as indicated in figure(1). The microstructure evolution also includes the increase of the concentration of the voids, namely the porosity defined as f. Moreover, power-law behavior will be assumed for the non-vacuous phase (matrix) labelled as 1, which reads as

$$U^{(1)}(\boldsymbol{\sigma}) = \frac{D_0\,\sigma_0}{n+1}\left(\frac{\sigma_{eq}}{\sigma_0}\right)^{n+1}, \quad n = \frac{1}{m} \quad (1)$$

where m is the strain-rate sensitivity, parameter such that $0 \leq m \leq 1$, σ_0 is the flow stress of the non-vacuous phase, D_0 is a reference strain-rate and σ_{eq} is the equivalent von-Mises stress. It is important to mention here that, although, the matrix phase, 1, is considered to be incompressible the overall behavior of the composite turns out to be compressible. In the above definition of the power-law potential when $n = 1$ the effective response of the material is linear, while for $n > 1$ the behavior becomes nonlinear. When the limit $n \to \infty$ is considered the material becomes rigid-perfectly plastic.

For comparison purposes the results derived with the use of the SOM are plotted together with results given by the *variational method* (denoted as VAR) of Ponte Castañeda (1991), the Gurson model (1977) (denoted as GUR) for two-dimensional cylindrical voids in a perfectly-plastic matrix, finite element calculations (FEM) (ABAQUS, 2004) of a unit cell with an isolated, initially, cylindrical void embedded in a nonlinear matrix which satisfies the relation (1). Moreover, estimates delivered by the study of high-rank sequentially laminated porous composites, denoted as LAM, making use of the work of deBotton and Hariton (2002), are used in estimating the yield surfaces of isotropic porous media for all nonlinearities. In the last method, the fields in the phases are computed exactly, while the fields in the porous phase are uniform since they are constructed by a laminating procedure. In both the SOM and the VAR case, the fields in the inclusion phase (void) are also assumed to be uniform (Hashin-Shtrikman estimates).

2 EFFECTIVE CONSTITUTIVE LAWS

The effective behavior of a composite material is defined as the instantaneous relation between the average stress, $\overline{\boldsymbol{\sigma}} = \langle\boldsymbol{\sigma}\rangle$, and the average strain-rate, $\overline{\boldsymbol{D}} = \langle\boldsymbol{D}\rangle$, where the triangular brackets serve to denote volume averages over the representative volume element Ω. The overall response of the medium will be a function of the internal variables characterizing the microstructure, labelled at this point as s_α. For the above defined class of

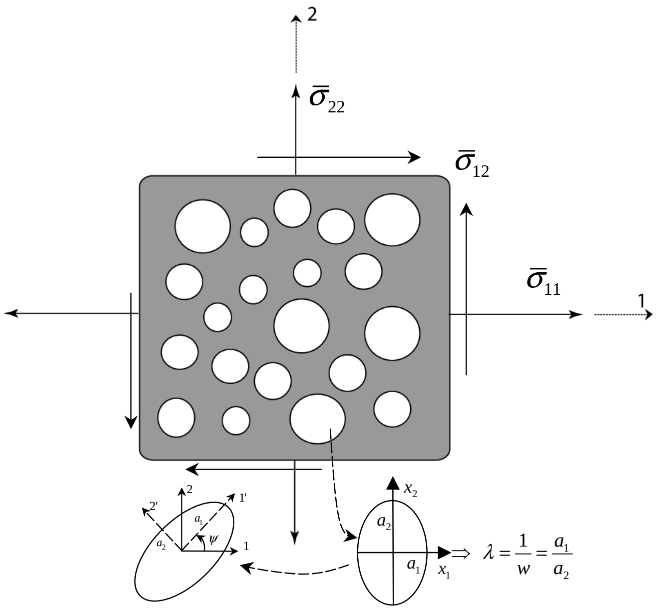

Figure 1: Image of the porous material with initially spherical voids. On the right lower corner the representative ellipsoid is shown. The aspect ratio is defined as shown.

materials, it is known from Hill (1963), that the effective constitutive behavior reads as,

$$\overline{\boldsymbol{D}} = \frac{\partial \widetilde{U}}{\partial \overline{\boldsymbol{\sigma}}}(\overline{\boldsymbol{\sigma}}; s_\alpha),$$
$$\widetilde{U}(\overline{\boldsymbol{\sigma}}; s_\alpha) = \inf_{\boldsymbol{\sigma}\,\epsilon\,\Im(\overline{\boldsymbol{\sigma}})} \langle U(\mathbf{x}, \boldsymbol{\sigma}) \rangle \qquad (2)$$

where $\Im(\overline{\boldsymbol{\sigma}}) = \{\boldsymbol{\sigma}, \mathrm{div}\boldsymbol{\sigma} = 0 \text{ in } \Omega, \langle\boldsymbol{\sigma}\rangle = \overline{\boldsymbol{\sigma}}\}$ is the set of statically admissible stresses that are consistent with the average stress condition $\langle \boldsymbol{\sigma} \rangle = \overline{\boldsymbol{\sigma}}$, and $\widetilde{U}(\overline{\boldsymbol{\sigma}}; s_\alpha)$ is the effective stress potential of the composite. The second-order method delivers estimates for the effective potential of a general N-phase composite, which takes the general form:

$$\widetilde{U}(\overline{\boldsymbol{\sigma}}) = \underset{\mathbf{M}_0^{(s)}}{\text{stat}} \{ \widetilde{U}_T(\overline{\boldsymbol{\sigma}}; \mathbf{M}_0^{(s)}) - \sum_{r=1}^{N} c^{(r)} V^{(r)}(\mathbf{M}_0^{(r)}) \}, \qquad (3)$$

where the $\widetilde{U}_T$ is the effective potential of a *linear comparison composite* (LCC) with the same microstructure as the nonlinear one and $V^{(r)}$ are the error functions defined in Ponte Castañeda (2002 a,b). The stationary operation in relation (3) consists in setting the partial derivative of the argument with respect to the fourth-order tensors $\mathbf{M}_0^{(s)}$ equal to zero.

3 APPLICATIONS

In this section, the SOM is applied initially in isotropic porous, power-law composites in order to deliver estimates for the yield surfaces of the effective medium. Comparisons are made with the available methods mentioned in the introduction. Then, the SOM is used to deliver estimates on the evolution of porosity under uniaxial tension loading for different values of the nonlinearity parameter n.

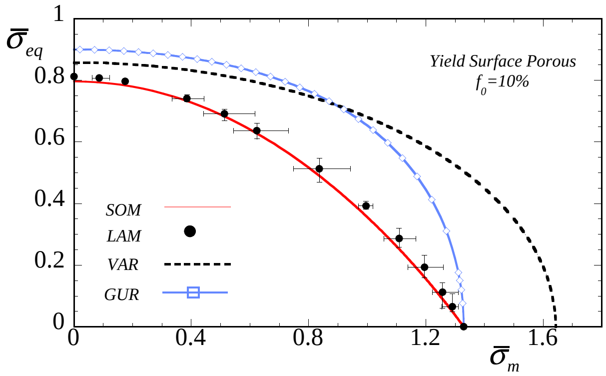

Figure 2: Yield surfaces of isotropic porous composite for initial porosity of $f_0 = 10\%$.

In figure (2) yield surfaces for $n \to \infty$ are shown. The SOM is in remarkable agreement with the estimate delivered by the LAM. In addition, both the SOM and the LAM exhibit a *corner* on the hydrostatic axis, which is a very crucial effect when the normal to that surface needs to be computed. Similar shape of yield functions were also observed

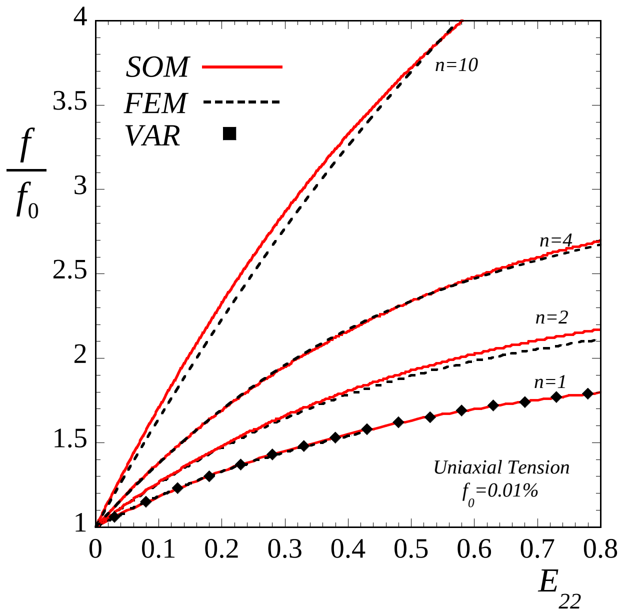

Figure 3: Evolution of porosity under uniaxial tension loading for various nonlinearities. Initial porosity of $f_0 = 0.01\%$ and initial aspect ratio $\lambda = 1$.

by Pastor and Ponte Castañeda (2002). It is also important to mention that except for the VAR method all the rest recover the analytical point in the pure hydrostatic limit

($\overline{\sigma}_{eq} \to 0$). However, despite the fact that the GUR estimate is exact in that limit, it does not capture the effect of the corner which is very important when the evolution of microstructure takes place. The GUR estimate also violates the variational bound, VAR, at low triaxialities.

Finally, the evolution of porosity for various nonlinearities, when the specimen is subjected to uniaxial tension loading, is shown in figure (3). It is evident that the SOM is in remarkable agreement with the FEM estimates for the evolving porosity, while it improves on the earlier estimates delivered by the VAR method which seriously underestimate the porosity evolution when the composite is highly nonlinear. In fact, the VAR method predicts the same behavior for all nonlinearities when it is plotted with respect to the applied strain.

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